PHASE MATCHING IN BBO FOR PARAMETRIC DOWNCONVERSION OF 405 NM LIGHT

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1. Anistropic Crystals

A "heralded" source of single photons can be realized with spontaneous parametric downconversion, a non-linear optical effect in which a photon incident on a non-linear optical crystal stimulates the emission of two new photons. The generated photon frequencies add up to the frequency of the incident photon. This process is shown schematically in Figure 1 where ω_p, the incident light ("pump" laser) spontaneously generates signal ω_s and the idler ω_i photons. This effect is very weak at the milliwatt light power levels typically used in this experiment, roughly one photon out of 10^{11} photons results in a conversion.

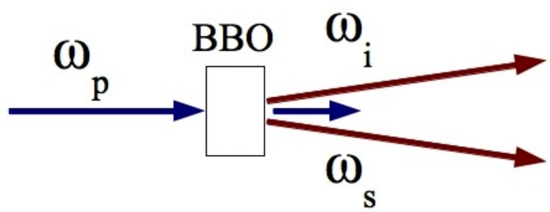

Figure 1. Spontaneous parametric down conversion

This process also satisfies momentum conservation in a scheme called phase matching between the laser light and the downconverted light. Before discussing phase matching in detail, it is instructive to consider light propagation in a uniaxial crystal, a class of optical crystals which includes BBO. We will assume that our coordinates are aligned with the crystal system such that

$$\vec{D} = \epsilon \vec{E} = \begin{pmatrix} \epsilon_{or} & 0 & 0 \\ 0 & \epsilon_{or} & 0 \\ 0 & 0 & \epsilon_{ex} \end{pmatrix} \vec{E}$$

where ϵ_{or} is the ordinary dielectric constant and ϵ_{ex} is the extraordinary dielectric constant. The optic axis points along the z-axis, which corresponds to the axis where propagation is independent of polarization (that is, $\epsilon_x = \epsilon_y = \epsilon_{or}$).

Herein, ϵ is assumed to be a matrix. In addition, assuming a non-magnetic material, $\mu = \mu_0$, Maxwel's equations combine to give

$$\nabla \times \nabla \times \vec{E} = -\mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Assume plane waves of the form

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} = \left(E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k}\right) e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

where $\vec{E}_0$ is a constant vector with components E_x, E_y, and E_z, and the direction of propagation is parallel to $\vec{k} = k_x \hat{\imath} + k_y \hat{\jmath} + k_z \hat{k}$.

This is a valid solution if $\vec{k}$ and ω satisfy the dispersion relation given by Maxwell's wave equation,

$$-\vec{k} \times \vec{k} \times \vec{E}_0 = k^2 \vec{E}_0 - \vec{k}(\vec{k} \cdot \vec{E}_0) = \mu_0 \omega^2 \epsilon \vec{E}_0$$

Since the crystal is anisotropic, we cannot assume $\vec{k} \cdot \vec{E}_0 = 0$ in all cases, which leaves us with the following three equations, one from each vector component

$$\begin{aligned} k^2 E_x - k_x(\vec{k} \cdot \vec{E}_0) &= \mu_0 \omega^2 \epsilon_{or} E_x \\ k^2 E_y - k_y(\vec{k} \cdot \vec{E}_0) &= \mu_0 \omega^2 \epsilon_{or} E_y \\ k^2 E_z - k_z(\vec{k} \cdot \vec{E}_0) &= \mu_0 \omega^2 \epsilon_{ex} E_z \end{aligned}$$

We can solve for each component of $\vec{E}_0$ in terms of $\vec{k} \cdot \vec{E}_0$,

$$E_x = \frac{k_x(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{or}} \quad E_y = \frac{k_y(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{or}} \quad E_z = \frac{k_z(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{ex}} \tag{1}$$

Combine these to form the dot product $\vec{k} \cdot \vec{E}$

$$k_x E_x + k_y E_y + k_z E_z = \frac{k_x^2(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{or}} + \frac{k_y^2(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{or}} + \frac{k_z^2(\vec{k} \cdot \vec{E}_0)}{k^2 - \mu_0 \omega^2 \epsilon_{ex}}$$

We can derive a dispersion relation by canceling the common factor $\vec{k} \cdot \vec{E}$. However, keep in mind that a second solution simply satisfies $\vec{k} \cdot \vec{E} = 0$, which is called the ordinary wave (more about this later). For now, consider the extraordinary wave solution where $\vec{k} \cdot \vec{E} \neq 0$,

$$1 = \frac{k_x^2}{k^2 - \mu_0\omega^2\epsilon_{or}} + \frac{k_y^2}{k^2 - \mu_0\omega^2\epsilon_{or}} + \frac{k_z^2}{k^2 - \mu_0\omega^2\epsilon_{ex}}$$

It is convenient to write the components of $\vec{k}$ in spherical coordinates with polar angle θ and azimuth ϕ, $k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$ and $k_z = k\cos\theta$, where the magnitude $\left|\vec{k}\right| \equiv k = \omega n/c$.

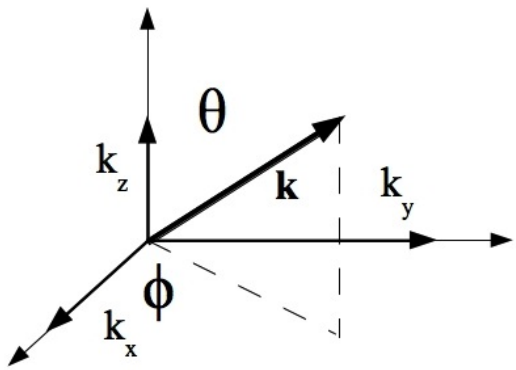

FIGURE 2. Propagation direction in spherical coordinates

We will also need the definitions for the ordinary refractive index $\mu_0\epsilon_{or} = (n_{or}/c)^2$ and the extraordinary $\mu_0\epsilon_{ex} = (n_{ex}/c)^2$. It is convenient to define the terms $k_{or} = \omega n_{or}/c$ and $k_{ex} = \omega n_{ex}/c$. Putting these forms of k into the dispersion relation results in

$$1 = \frac{k^2\sin^2\theta}{k^2 - k_{or}^2} + \frac{k^2\cos^2\theta}{k^2 - k_{ex}^2}$$

A little more algebra gives the solution for $k^2(\theta)$,

$$k^2(\theta) = \frac{k_{or}^2 k_{ex}^2}{k_{or}^2\sin^2\theta + k_{ex}^2\cos^2\theta} \tag{2}$$

or in the same form as presented in Galvez, et al [1]

$$n^2(\theta) = \frac{n_{or}^2 n_{ex}^2}{n_{or}^2\sin^2\theta + n_{ex}^2\cos^2\theta} = \frac{1}{(\sin\theta/n_{ex})^2 + (\cos\theta/n_{or})^2} \tag{3}$$

As mentioned, this solution gives the index for the extraordinary wave and corresponds to an electric field pointing in the k-z plane. There is another solution for plane waves, called the ordinary wave, where $n = n_{or}$ and the electric field points in the x-y plane. This can be shown by analyzing the components of $\vec{E}$, which is done as an appendix.

In summary, for a given direction of propagation, there are two polarizations and corresponding refractive index values.

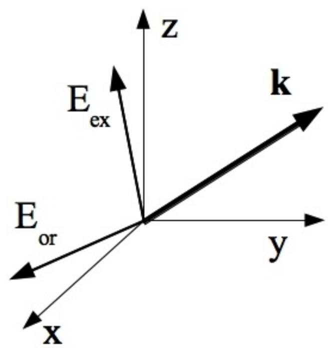

FIGURE 3. Ordinary and extraordinary polarizations.

The ordinary wave is polarized in the x-y plane and has refractive index n_{or}. The extraordinary wave is polarized in the k-z plane and has a refractive index given by (3).

2. PHASE MATCHING

Parametric downconversion is typically characterized by the wave-mixing relation (energy conservation)

$$\hbar\omega_p = \hbar\omega_i + \hbar\omega_s,$$
$$\omega_p = \omega_i + \omega_s$$

where ω_p is the angular frequency of the incident beam, which is referred to as the pump, and ω_i and ω_s are the angular frequencies of the signal and idler beams. The phase matching condition is

$$\vec{k}_p = \vec{k}_i + \vec{k}_s \tag{4}$$

where

$$\left|\vec{k}_p\right| = \frac{n_p(\omega_p)\omega_p}{c} \qquad \left|\vec{k}_i\right| = \frac{n_i(\omega_i)\omega_i}{c} \qquad \left|\vec{k}_s\right| = \frac{n_s(\omega_s)\omega_s}{c}$$

and the refractive index values depend on frequency and polarization. These two relations, energy and momentum conservation, cannot in general be satisfied in isotropic materials due to normal dispersion. In particular, the refractive index increases with frequency so that $n(\omega_p) > n(\omega_s)$ and $n(\omega_p) > n(\omega_i)$. The combined momentum of the signal and idler is always less than the momentum of the pump.

In the last section, we showed how light propagation in a birefringent crystal exhibits a polarization dependent refractive index. This can be used to compensate for the natural dispersion and achieve phase matching. In the case of BBO, beta-barium-borate, the refractive index values depend on wavelength according to a phenomenological Sellmeier equation

$$n(\lambda) = \sqrt{A + \frac{B}{\lambda^2 + C} + D\lambda^2} \tag{5}$$

where the coefficients for the ordinary and extraordinary refractive index are measured [2]

n	A	$B(\mu\mathrm{m}^2)$	$C(\mu\mathrm{m}^2)$	$D(\mu\mathrm{m}^{-2})$
n_{or}	2.7359	0.01878	-0.01822	-0.01354
n_{ex}	2.3753	0.01224	-0.01667	-0.01516

Figure 4 shows a Mathematica plot of $n_{or}(\lambda)$ and $n_{ex}(\lambda)$ for BBO

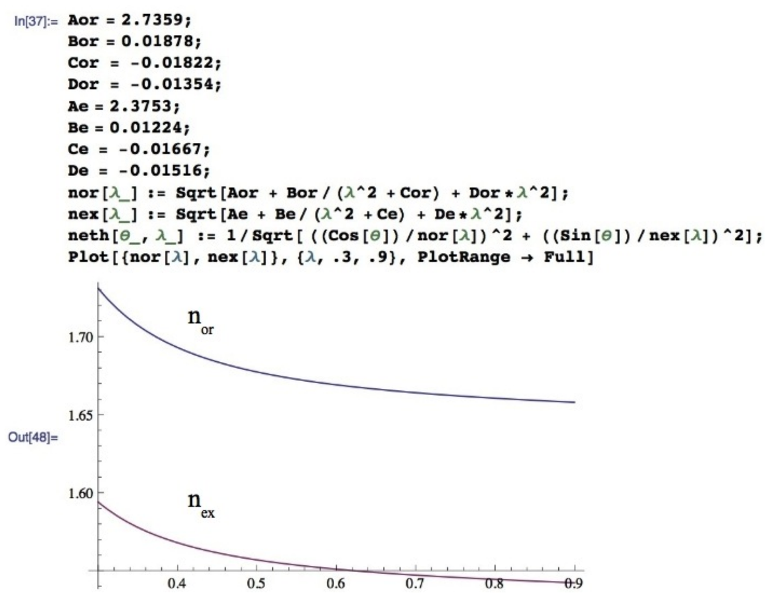

FIGURE 4. Ordinary and extraordinary refractive index plots for BBO as a function of light wavelenth.

At any given wavelength, the ordinary wave propagates with a larger refractive index than the extraordinary $n_{or} > n_{ex}$ (unless $\theta = 0$, then they are the same). Phase matching can be achieved if the signal and idler waves are polarized along the y-axis, with refractive index values $n_{or,\lambda=0.810\mu\text{m}} = 1.66026$ and the pump beam is polarized in the x-z plane with refractive index varying according to (3) between $n_{or,\lambda=0.405\mu\text{m}} = 1.69189$ and $n_{ex,\lambda=0.405\mu\text{m}} = 1.56712$. This configuration, where the signal and idler waves have the same polarization and are perpendicular to the pump polarization, is called Type I phase matching.

The Figure 5 shows Type I geometry for propagation in the x-z plane ($\phi = 0$) at an angle θ to the z-optical axis. The y-axis points out of the page and polarization vectors are represented by dots. Two cases of Type I phase matching are calculated, collinear and non-collinear.

Collinear phase matching requires that $n(\theta)_{\lambda=0.405\mu\text{m}} = n_{or,\lambda=0.810\mu\text{m}}$ with the result (using the same Mathematica functions as in Figure 4)

FindRoot[neth[θ, 0.405] = = nor[0.810], {θ, .5}]

{$\theta \to$ 0.502932 }

or $\theta_{coll} = 28.8°$.

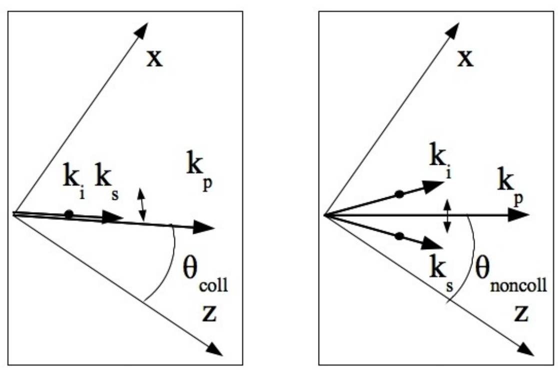

FIGURE 5. Type I phase matching geometry for collinear and non-collinear parametric generation. In non-collinear phase matching, the angle between the signal or idler propagation and the pump propagation is taken to be 3°

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The non-collinear geometry used in the single photon labs has an angle of 3° between each of the downconverted beams and the pump beam. This adds a $\cos 3°$ factor to the expression for phase matching,

$$\left|\vec{k}_p\right| = \left|\vec{k}_s\right| \cos 3° + \left|\vec{k}_i\right| \cos 3° \Longrightarrow n(\theta)_p\omega_p = \cos 3° n_s \frac{1}{2}\omega_p + \cos 3° n_i \frac{1}{2}\omega_p$$

where it is assumed that $\omega_s = \omega_i = \omega_p/2$. This gives a slightly larger phase matching angle,

FindRoot[neth[θ, 0.405] = = Cos[3 Pi / 180] nor[0.810], {θ, .5}]

{$\theta \to 0.523076$ }

or $\theta_{noncoll} = 30.0°$. This is slightly larger than the value given in [3], perhaps the refractive index values are slightly different than Kato reports in [2].

3. Appendix: Field calculations

First, the math is clearer if we choose to propagate in the x-z plane. Since there is symmetry in x-y, there is no loss of generality if we choose our coordinates such that $k_y = 0$.

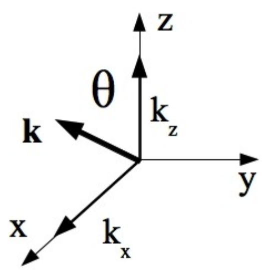

Figure 6. Propagation direction in spherical coordinates

Consider the extraordinary wave with solution for k given by (2), which assumes that $\vec{k} \cdot \vec{E} \neq 0$. The field equations given in (1) are

$$E_x = \frac{k_x(\vec{k} \cdot \vec{E}_0)}{k^2 - k_{or}^2} = \frac{k \sin\theta(\vec{k} \cdot \vec{E}_0)}{k^2(1 - k_{or}^2/k^2)} = \frac{\sin\theta(\vec{k} \cdot \vec{E}_0)}{k(1 - k_{or}^2((\cos\theta/k_{or})^2 + (\sin\theta/k_{ex})^2))}$$

$$= \frac{\sin\theta(\vec{k} \cdot \vec{E}_0)}{k(1 - \cos^2\theta - k_{or}^2 \sin^2\theta/k_{ex}^2)} = \frac{\vec{k} \cdot \vec{E}_0}{k \sin\theta} \frac{k_{ex}^2}{k_{ex}^2 - k_{or}^2}$$

$$E_y = \frac{k_y(\vec{k} \cdot \vec{E}_0)}{k^2 - k_{or}^2} = 0$$

and

$$E_z = \frac{k_z(\vec{k} \cdot \vec{E}_0)}{k^2 - k_{ex}^2} = \frac{\vec{k} \cdot \vec{E}_0}{k \cos\theta} \frac{k_{or}^2}{k_{or}^2 - k_{ex}^2}$$

Clearly, the field is polarized in the x-z plane (also the k-z plane as mentioned earlier) and the ratio of field components is

$$\frac{E_x}{E_z} = -\frac{k_{ex}^2 \cos\theta}{k_{or}^2 \sin\theta}$$

The ordinary wave is where $\vec{k} \cdot \vec{E} = k_x E_x + k_z E_z = 0$ and the only non trivial solution is where $E_x = E_z = 0$, $E_y \neq 0$, and $k = k_{or}$.

References

[1] E. J. Galvez, C. H. Holbrow, M. J. Pysher, J. W. Martin, N. Courtemanche, L. Heilig, and J. Spencer, *Interference with correlated photons: Five quantum mechanics experiments for undergraduates*, Amer. J. Phys., **73**, 127 (2005).

[2] K. Kato, *Second-harmonic generation to 2048 Å in β-BaB_2O_4*, IEEE J. Quantum Electron., **QE-22**, 1013 (1986).

[3] Dietrich Dehlinger and M. W. Mitchell, *Entangled photon apparatus for the undergraduate laboratory*, Amer. J. Phys., **70**, 898 (2002).

[4] J. J. Thorn, M. S. Neel, V. W. Donato, G. S. Bergreen, R. E. Davies, and M. Beck, *Observing the quantum behavior of light in an undergraduate laboratory*, Amer. J. Phys., **72**, 1210 (2004).

[5] Benjamin Plotkin-Swing and Bevan Gerber-Siff, *Creation of single photons with spontaneous parametric down conversion*, Phys 82 laboratory report, Swarthmore College (2008).