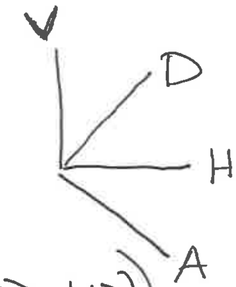
Math behind the process of zeroing ϕ:



$$\frac{1}{\sqrt{2}}\left(|H\rangle|H\rangle + e^{i\phi}|V\rangle|V\rangle\right)$$

$$= \frac{1}{2\sqrt{2}}\left(\left(|D\rangle+|A\rangle\right)\left(|D\rangle+|A\rangle\right) + e^{i\phi}\left(|D\rangle-|A\rangle\right)\left(|D\rangle-|A\rangle\right)\right)$$

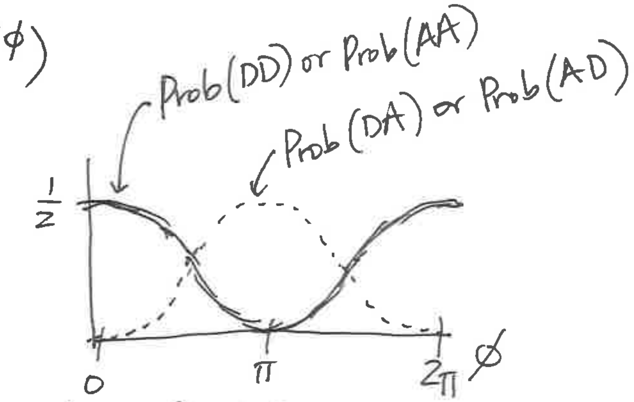
$$= \frac{1}{2\sqrt{2}}\left[\left(1+e^{i\phi}\right)\left(|D\rangle|D\rangle+|A\rangle|A\rangle\right) + \left(1-e^{i\phi}\right)\left(|D\rangle|A\rangle+|A\rangle|D\rangle\right)\right]$$

$$\text{Prob}(AA) = \text{Prob}(DD) = \left(\frac{1}{2\sqrt{2}}\right)^2\left(1+e^{i\phi}\right)\left(1+e^{-i\phi}\right)$$

$$= \frac{1}{8}\left(2+2\cos\phi\right)$$

$$= \frac{1}{2}\cos^2\left(\frac{\phi}{2}\right)$$

$$\text{Prob}(DA) = \text{Prob}(AD) = \frac{1}{2}\sin^2\left(\frac{\phi}{2}\right)$$



Average value of ϕ is zero when Prob(DD) is maximized and Prob(DA) is minimized.

Math behind the entanglement purity measurement:

We assume we are dealing with state $\frac{1}{\sqrt{2}}\left(|H\rangle|H\rangle+|V\rangle|V\rangle\right)$ with probability p and state $\{50\%\ |H\rangle|H\rangle,\ 50\%\ |V\rangle|V\rangle\}$ w/ prob $(1-p)$.

Here p is the purity of the entangled state.

With probability $1-p$, we will get equal rates of DD, AA, DA, and AD.

With probability p, we will get DD and AA pairs but <u>no</u> DA or AD pairs.

If we measure N_{tot} total pairs, we can expect $N_{DA} = (1-p)\frac{N_{tot}}{4} = N_{AD}$

$$N_{DD} = (1-p)\frac{N_{tot}}{4} + p\frac{N_{tot}}{2} = N_{AA}$$

$$\ldots \text{so} \left\{\begin{array}{l} N_{DA} = N_{AD} = \left(\frac{1}{4}-\frac{p}{4}\right)N_{tot} \\ N_{DD} = N_{AA} = \left(\frac{1}{4}+\frac{p}{4}\right)N_{tot} \end{array}\right\}$$

$$\text{Thus } \frac{N_{DD}-N_{DA}}{N_{DD}+N_{DA}} = \frac{\frac{p}{2}N_{tot}}{\frac{1}{2}N_{tot}} = p$$

If we measure the rates of DD and DA coincidences well, we can calculate the purity of the entangled state under our assumptions.